(i) Printed Pages: 3 Roll No. ....

(ii) Questions : 8

Sub. Code : | 0 | 4 | 4 | 3 |

Exam. Code : | 0 | 0 | 0 | 5 |

B.A./B.Sc. (General) 5th Semester

(2122)

MATHEMATICS

Paper–I : Analysis–I

Time Allowed : Three Hours] [Maximum Marks : 30

Note :—Attempt **five** questions in all, selecting at least **two** questions from each Unit.

UNIT–I

1. (a) Show that set of integers is countable.

   (b) Let $f(x) = \frac{1}{x^2}$ on [1, 4]. Find L(p, f) by dividing [1, 4] into three equal sub-intervals. 3+3=6

2. (a) Prove that every continuous function on closed interval is Riemann integrable.

   (b) If f is R-integrable on [a, b] and $|f(x)| \le k \; \forall \; x \in [a, b]$ then prove that :

$$\left|\int_a^b f(x)\,dx\right| \le k(b-a)$$ 3+3=6

3. (a) Show that $\frac{1}{3\sqrt{2}} \le \int_0^1 \frac{x^2}{\sqrt{1+x^2}}\,dx \le \frac{1}{3}$

(b) Give an example of a bounded function "f" defined on a closed interval such that |f| is R-integrable but f is not. 3+3=6

4. (a) Prove that $\int_{-1}^{\infty} \frac{x+1}{(x+2)^6}\,dx = \frac{1}{20}$.

(b) Use $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ to prove $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

3+3=6

UNIT–II

5. (a) Test for convergence of integral $\int_0^{\infty} \frac{\sin^2 x}{x^2}\,dx$.

(b) Examine for convergence $\int_0^{\infty}\left(\frac{1}{x} - \frac{1}{\sinh x}\right)\frac{dx}{x}$. 3+3=6

6. (a) Use Dirichlet's test to show $\int_0^{\infty} \frac{\sin x}{x}\,dx$ is convergent at ∞.

(b) State and prove Abel's test for convergence of Improper integral. 3+3=6

7. (a) Discuss the convergence of the integral

$$\int_1^2 \frac{dx}{(x-1)^{1/2}(2-x)^{1/3}}.$$

(b) Use Frullani Theorem and prove that :

$$\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx = \log\left(\frac{b}{a}\right) \text{ where } a > 0,\ b > 0.$$

3+3=6

8. (a) If $|a| < 1$ then evaluate $\int_0^\pi \frac{\log(1 + a\cos x)}{\cos x} dx$.

(b) Using $\int_0^\infty \frac{dx}{x^2 + a} = \frac{\pi}{2\sqrt{a}}$, prove that :

$$\int_0^\infty \frac{dx}{(x^2 + a)^{n+1}} = \frac{\pi}{2} \frac{1.3.5......(2n-1)}{2.4.6......(2n)} \cdot \frac{1}{a^{n+\frac{1}{2}}}$$

3+3=6