(i) Printed Pages : 2   Roll No. ....

(ii) Questions   : 8   Sub. Code : | 0 | 4 | 4 | 4 |
                         Exam. Code : | 0 | 0 | 0 | 5 |

B.A./B.Sc. (General) 5ᵗʰ Semester
(2122)
MATHEMATICS
Paper–II (Modern Algebra)

Time Allowed : Three Hours]   [Maximum Marks : 30

Note :— Attempt **five** questions in all, choosing at least **two** questions from each Unit. All questions carry equal marks.

UNIT–I

1. (a) Find the orders of elements of Quaternions group Q_8. Is Q_8 cyclic ? Justify.

   (b) Let G be a semi-group and a, b $\in$ G. Prove that G is a group if and only if both the cancellation laws hold in G.   2,4

2. (a) If H and K are two subgroups of a group G, then prove that HK is subgroup of G iff HK = KH.

   (b) Let G be a group and a, b $\in$ G. Is $O(ab) = O(a).O(b)$, in general ? Justify.   4,2

3. (a) Define index of a subgroup H in a group G. Prove that $O(G) = O(H)[G : H]$.

   (b) Let G be a group such that G/Z(G) is cyclic, where Z(G) is centre of the group G, then prove that G is abelian.   3,3

4.　(a)　Prove that there are only two groups of order 6.

　　(b)　Compute $p^{-1}qp$ where $p = (1, 3, 5)(2, 4)$ and $q = (1, 4, 2, 5)(6, 3)$. 　　　　3,3

UNIT–II

5.　(a)　Define division ring. Prove that every division ring is a simple ring.

　　(b)　Let E be the ring of even integers. Prove that <4> is a maximal ideal in E. 　　　　3,3

6.　(a)　Prove that a commutative ring with unity is a field iff it does not have any proper ideal.

　　(b)　Prove that in the ring of integers Z, the ideal $<m> = mZ = \{mn : n \in Z\}$ is a prime ideal if and only if m is prime number. 　　　　3,3

7.　(a)　State and prove fundamental theorem of ring homomorphism.

　　(b)　Is the field R of real numbers isomorphic to the field C of complex numbers ? Justify. 　　　　4,2

8.　(a)　If R is an integral domain, then prove that R[x] is also integral domain.

　　(b)　What are the units of the polynomial ring $Z_7[x]$? 　　　　3,3